Kuwait University College of Science Department of Mathematics	Calculus I (Course No. 0410101) Exam 2	Date: 07/12/2011 Time: 18:30 - 20:00 Fall, 2011/2012

Mobiles and Calculators are Not Allowed

Show all your work to get credit

Question 1 (4 points): Find an equation for the tangent line to the curve $y = 2 + \sin(xy)$ at $x = 0$.

Question 2 (4 points): Two people start walking from the same point at the same time. One walks east at a rate of 8 km/hr and the other walks south at a rate of 6 km/hr. How fast is the distance between them changing after half an hour?

Question 3 (4 points): Use linear approximation to estimate the number $(0.94)^{1/3}$.

Question 4 (4 points): Let $h(x)$ be a differentiable function satisfying

$$h(15) = 123 \text{ and } h'(x) \leq 10 \text{ for all real numbers } x.$$

Show that $h(6) \geq 33$.

Question 5 (2+2+2+3 points): Let $g(x) = \dfrac{2 + x - x^2}{(x-1)^2}$. Given that $g'(x) = \dfrac{x-5}{(x-1)^3}$ and $g''(x) = \dfrac{14 - 2x}{(x-1)^4}$:

(a) Find all asymptotes, if any.

(b) Find the intervals on which the graph of g is increasing and the intervals on which the graph of g is decreasing. Also, find the local extrema of g, if any.

(c) Find the intervals on which the graph of g is concave upward and the intervals on which the graph of g is concave downward. Also, find the points of inflection, if any.

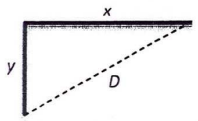
(d) Sketch the graph of g. Show on your graph all asymptotes, vertical and horizontal tangents, intercepts, inflection points, and all local and absolute extrema, if any.

Kuwait University College of Science Department of Mathematics	Calculus I (0410101) Exam 2 (Solution)	Date: 07/12/2011 Time: 18:30 - 20:00 Fall, 2011/2012

Qustion 1 (4 points): $y = 2 + \sin(xy) \rightarrow y' = \cos(xy)(y + xy') \rightarrow \boxed{y' = \dfrac{y\cos(xy)}{1 - x\cos(xy)}}$.

$\boxed{x = 0 \rightarrow y = 2 + \sin 0 = 2} \rightarrow \boxed{m = y'|_{(0,2)} = 2}$. → Equ. of Tangent: $\boxed{y - 2 = 2(x - 0) \rightarrow y = 2x + 2.}$

Qustion 2 (4 points): Given: $\dfrac{dx}{dt} = 8$ km/hr, $\dfrac{dy}{dt} = 6$ km/hr.



$D = \sqrt{x^2 + y^2} \rightarrow \boxed{\dfrac{dD}{dt} = \dfrac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2 + y^2}} = \dfrac{8x + 6y}{\sqrt{x^2 + y^2}}}$.

After 1/2 hr: $\boxed{x = 4 \text{ km}, y = 3 \text{ km}} \rightarrow \boxed{\dfrac{dD}{dt} = \dfrac{32 + 18}{\sqrt{25}} = 10 \text{ km/hr}}$

Qustion 3 (4 points): Let $g(x) = \sqrt[3]{x} \rightarrow g'(x) = \dfrac{1}{3}x^{-2/3} \rightarrow g(1) = 1$ and $g'(1) = 1/3 \rightarrow$

$\boxed{g(x) = \sqrt[3]{x} \approx 1 + \frac{1}{3}(x - 1) = \dfrac{x + 2}{3}} \rightarrow \boxed{(0.94)^{1/3} = g(0.94) \approx \dfrac{0.94 + 2}{3} = 0.98}$.

Qustion 4 (4 points): $M.V.Th.$ → there exists c in $(6, 15)$ such that

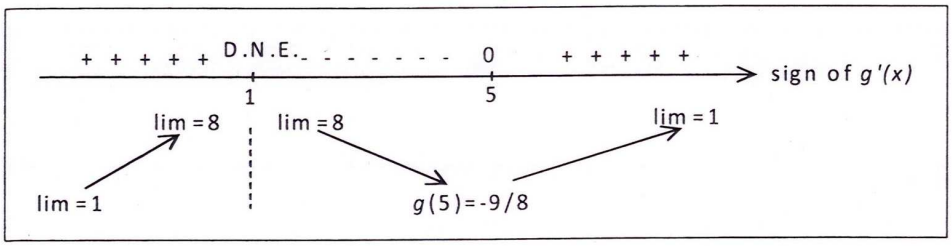
$$\boxed{h(15) - h(6) = h'(c)(15 - 6)} \rightarrow 123 - h(6) = 9h'(c) \rightarrow h(6) = 123 - 9h'(c).$$

But $h'(c) \leq 10$. Therefore $\boxed{h(6) \geq 123 - 9(10) = 33}$.

Qustion 5 (2+2+2+3 points): Let $g(x) = \frac{2+x-x^2}{(x-1)^2}$. Given: $g'(x) = \frac{x-5}{(x-1)^3}$ and $g''(x) = \frac{14-2x}{(x-1)^4}$.

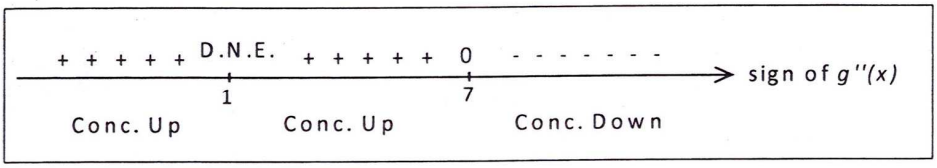
(a) $\lim_{x\to 1} g(x) = \frac{2}{0^2} = +\infty \to x = 1$ is a V.A. | $\lim_{x\to\pm\infty} g(x) = -1 \to y = -1$ is a H.A. as $x \to \pm\infty$. | (No Slant A.)

(b) $g'(x)$ D.N.E. $\to x = 1 \notin Domain(g) = R\backslash\{1\}$. | $g'(x) = 0 \to x = 5$ (Critical number)



$\therefore$ Since g is cont. at $x = 5$, $g(5) = -9/8$ is a local minimum (or Abs. Min.)

(c) $g''(x)$ D.N.E. $\to x = 1 \notin Domain(g) = R\backslash\{1\}$. | $g''(x) = 0 \to x = 7$.



$\therefore$ Since g is cont. at $x = 7$ $(7, -10/9)$ is an inflection point.

(d) **x-intercepts**: $g(x) = 0 \to x = -1, 2$. | y-intercept: $y = g(0) = 2$.